Jour. Ind. Soc. Ag. Statistics
Vol. XLI, No. 3 (1989), pp. 316-317

A NOTE ON NARAIN'S NECESSARY CONDITION IN SAMPLING

J. N. K. RAO
Carleton University, Ottawa, Canada
(Received : July, 1989)

SUMMARY

P. Singh [2] claimed to have "disproved" Narain's [1] necessary condition for without replacement sampling to have smaller variance than with replacement sampling, This note shows that his claim is false.

Keywords : Sampling with replacement, Sampling without replacement, Comparison of variances,

Narain [1] has shown that a necessary condition for without replacement sampling to have smaller variance than with replacement sampling is given by

$$\pi_{ij} \leq \frac{2(n-1)}{n} \pi_i \pi_j \quad \text{for all } i \neq j = 1, \ldots, N. \tag{1}$$

Here $\pi_i (= np_i)$ and π_{ij} denote the first order and second order inclusion probabilities in without replacement sampling, p_i is the selection probability associated with the ith propulation unit ($i = 1, \ldots, N$) and n is the sample size. The meaning of Narain's result is that if the variance of estimated total $\hat{Y}_{wr}$ in without replacement sampling is smaller than the variance of estimated total $\hat{Y}_{wr}$ in with replacement sampling for all $y = (y_1, \ldots, y_N)'$, then (1) holds.

P. Singh [2] claims to have disproved this result by constructing a particular design under which var $(\hat{Y}_{wr}) <$ var $(\hat{Y}_{wr})$ for one *particular* y and at the same time condition (1) is violated for at least one pair of units (i, j). Singh's example, however, in no way contradicts Narain's result since the latter result requires var $(\hat{Y}_{wr}) \leq$ var $(\hat{Y}_{wr})$ for all y. To

disprove Narain's result, Singh will have to show analytically that his particnlar design gives var $(\hat{\bar{Y}}_{w_{1r}}) \leqslant$ var $(\hat{\bar{Y}}_{wr})$ for *all y*, which is obviously impossible.

REFERENCES

[1] Narain, R. D. (1951) : On sampling without replacement with varying probabilies, *Jour. Ind. Soc. of Ag. Stat.* 3 : 169-174.

[2] Singh, P. (1988) : On Narain's necessary condition in sampling, *Jour. Ind. Soc. of Ag. Stat.* 40 : 218-222.